SOLUTIONS AND PROBLEMS.

ARITHMETIC.

Balls in a box, Nos. 120 and 130 ..... 14, 70, 103, 189–190

Bought 150 head of stock for $300, paying for each kind a certain price. Find number of each kind, No. 126 135–136

Building and Loan Association Problem, No. 127 167–168

Circles, three equal, No. 125 103–104

Clock Problem, No. 124 70–72

Close of the Nineteenth Century 292

Figure 7 between hour and minute hands, No. 128 167–168

Frustrum, conical, to be divided into n parts, No. 131 223–224

If m cents be interest for M cents for p days, find yearly rate, No. 123 38

If 6 is half of 10, what part of 20 is 12? 292

Marble, Iron, Copper, Silver, Gold balls. Problem (in rhyme), No. 129 168–169

Road, 60 feet wide, crosses a square acre of land, West line passes through southwest corner of the land, east line through northeast corner. What fraction of the land in the road? No. 132 258–260

Rule for finding the number of board feet in round timber. How obtained, 292

Suppose 10% traction stock is 20% better in the market than 5% mining stock; if my income is $500. from each, how much have I paid, the investment bringing $6\frac{2}{3}$%? No. 122 38

Three men with their sons, No. 121 14–15

ALGEBRA.

Farmer had 2080 lbs. of grain at the depot, etc., vid. problem No. 97 17–18

Field to be reaped by A and B for 90s, etc., vid problem No. 98 39–40

Given equation $x^m+p_1x^{m-1}+\ldots p_m$ freed from multiple roots. Prove discriminant positive or negative according as number of pairs of complex roots is even or odd. No. 103 137–138

Given $xyz=18$, $x^2+y^2+z^2=33$, $(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)=6561$. No. 107 224–225

Prize Problem, $x^2+y=2$, $x+y^2=6$. No. 4 169–172

Prove $\Sigma(n)+\frac{n}{2!}\Sigma(n^2)+\frac{n(n-1)}{3!}\Sigma(n^3)+\ldots=(n+1)^2-n$. No. 101 104–107

Put down any number of £, s, and d under £11, etc., vid. problem No. 110 261–263

Seven persons to be seated at a round table. No. 99 72–75

Solve $a^{\log(a^x)}=m^{\log m}$. No. 105 172

Solve $\frac{x^2+x}{y^2+y}=a$, $\frac{x^2+y}{y^2+x}=b$. No. 106 190–192

Solve $\frac{1}{\sqrt{(x+1)}}+\frac{1}{\sqrt{(x-1)}}=\frac{1}{\sqrt{(x^2-1)}}$. No. 109 260–261

GEOMETRY.

Bisectors, two, of a triangle being equal, prove triangle isosceles. No. 133 226–228

Conic passing through all foci of a conic is a rectangular hyperbola. No. 124 42–43

Construct triangle, base, median to the base, and difference of base angles being given. No. 136 264–266

Cone, right, has its vertex in horizontal plane, its axis perpendicular to the plane, and string wound spirally on cone, etc. No. 90 296–300

Draw circle, cutting two circles orthogonally. No. 132 193–194

Equation to three points of conjugate diameters of an ellipsoid. No. 127 107–108

SOLUTIONS AND PROBLEMS.

ARITHMETIC.

Balls in a box, Nos. 120 and 130 ..... 14, 70, 103, 189–190

Bought 150 head of stock for $300, paying for each kind a certain price. Find number of each kind, No. 126 135–136

Building and Loan Association Problem, No. 127 167–168

Circles, three equal, No. 125 103–104

Clock Problem, No. 124 70–72

Close of the Nineteenth Century 292

Figure 7 between hour and minute hands, No. 128 167–168

Frustrum, conical, to be divided into n parts, No. 131 223–224

If m cents be interest for M cents for p days, find yearly rate, No. 123 38

If 6 is half of 10, what part of 20 is 12? 292

Marble, Iron, Copper, Silver, Gold balls. Problem (in rhyme), No. 129 168–169

Road, 60 feet wide, crosses a square acre of land, West line passes through southwest corner of the land, east line through northeast corner. What fraction of the land in the road? No. 132 258–260

Rule for finding the number of board feet in round timber. How obtained, 292

Suppose 10% traction stock is 20% better in the market than 5% mining stock; if my income is $500. from each, how much have I paid, the investment bringing $6\frac{2}{3}$%? No. 122 38

Three men with their sons, No. 121 14–15

ALGEBRA.

Farmer had 2080 lbs. of grain at the depot, etc., vid. problem No. 97 17–18

Field to be reaped by A and B for 90s, etc., vid problem No. 98 39–40

Given equation $x^m+p_1x^{m-1}+\ldots p_m$ freed from multiple roots. Prove discriminant positive or negative according as number of pairs of complex roots is even or odd. No. 103 137–138

Given $xyz=18$, $x^2+y^2+z^2=33$, $(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)=6561$. No. 107 224–225

Prize Problem, $x^2+y=2$, $x+y^2=6$. No. 4 169–172

Prove $\Sigma(n)+\frac{n}{2!}\Sigma(n^2)+\frac{n(n-1)}{3!}\Sigma(n^3)+\ldots=(n+1)^2-n$. No. 101 104–107

Put down any number of £, s, and d under £11, etc., vid. problem No. 110 261–263

Seven persons to be seated at a round table. No. 99 72–75

Solve $a^{\log(a^x)}=m^{\log m}$. No. 105 172

Solve $\frac{x^2+x}{y^2+y}=a$, $\frac{x^2+y}{y^2+x}=b$. No. 106 190–192

Solve $\frac{1}{\sqrt{(x+1)}}+\frac{1}{\sqrt{(x-1)}}=\frac{1}{\sqrt{(x^2-1)}}$. No. 109 260–261

GEOMETRY.

Bisectors, two, of a triangle being equal, prove triangle isosceles. No. 133 226–228

Conic passing through all foci of a conic is a rectangular hyperbola. No. 124 42–43

Construct triangle, base, median to the base, and difference of base angles being given. No. 136 264–266

Cone, right, has its vertex in horizontal plane, its axis perpendicular to the plane, and string wound spirally on cone, etc. No. 90 296–300

Draw circle, cutting two circles orthogonally. No. 132 193–194

Equation to three points of conjugate diameters of an ellipsoid. No. 127 107–108

Given $F=\triangle^{n-1}\div(n-1)!\ \triangle_1.\triangle_2.\ldots\triangle_n$, etc., vid problem. No. 128 108–109
Hyperbola, touching four sides of an inscribed quadrilateral. If one focus lies on the circle, the other will lie on the circle, too. No. 135 264
Locus of a point on ellipsoid, the tangent plane at that point at distance f from center. No. 125 75–76
Medians, point of intersection of being G, prove $AB^2+BC^2+CA^2=3(GA^2+GB^2+GC^2)$. No. 123 41–42
Perpendiculars, dropped from vertices of a regular polygon upon any diameter. Prove sum on one side equals sum on other side. No. 122 ..... 40–41, 75
Points x, y, z divide strokes c—b, a—c, b—a, in same ratio r, and triangles x, y, z and a, b, c are similar, either $r=1$, or both triangles are equilateral. No. 130 172–173
Propositions on the regular dodecahedron 293–295
Prove $\lambda+\mu\omega+\nu\omega^2$ represents points of a quilt formed by regular hexagons. No. 131 173–174
Quadrilateral $ABCD$, circumscribing a circle. Line joining the middle points of diagonals AB and CD passes through center. No. 134 ..... 229–230, 295
Show that on no point of an ellipse will circle of curvature pass through center, eccentricity $<\frac{1}{2}\sqrt{2}$. No. 129 138–139
Through any fixed point O draw two straight lines at right angles. Let one line cut a given circle at Q, the other at R. Find locus of foot of perpendicular from O upon the chord QR. No. 126 77–78

CALCULUS.

Area of loop $r^2\cos\theta=a^2\sin^3\theta$. No. 91 19–20
A 24-inch joint of 6-inch stove pipe is compressed at one end to make it fit over an elliptical opening in a stove, etc., vid. problem. No. 101 231–232
Axes of three equal right cylinders intersect at right angles. Find common volume. No. 99 194
Cylinders, two intersecting. Find common volume. No. 92 20–22
Differential equation $(1-x^2)\frac{\partial^2 y}{\partial^2 x}-x\frac{\partial y}{\partial x}+y=2x$. No. 93 44–47
Epicycloid and hypocycloid. No. 98 139–140
If $f'(x)=\int f(x)\partial x$, find $f(x)$. No. 96 80
Minimum isosceles triangle about ellipse. No. 94 47
Park, elliptical, surrounded by wall. Horse fastened by a rope to wall; find area grazed over. No. 103 267–268
Prolate spheroid pierced by auger. Find volume removed. No. 97 109–110
Right cone has its vertex at focus of a paraboloid. Find common volume. No. 102 266–267
Ship, sailing northeast all the time. Find distance at given latitude and given longitude. No. 95 79–80
Volume, bounded by surface, generated by circumference of a circle, whose diameter is hypotenuse of a right triangle. No. 100 194–195

MECHANICS.

Assuming that velocity is proportionate to distance described from state of rest. Can a body start in motion, and if so, what is the initial acceleration? No. 95 82
Bow of a boat, inclined at a certain angle. When in motion the water was found to rise b inches on the bow. Find velocity of boat. No. 91 24
Fall from Heaven of the impious spirits. No. 87, ... 80–82, 110

Moments, principal, of a parallelogram. No. 94 and 97 ..... 48–49, 195
Particle slides down a parabola. Find initial velocity so that it may leave curve at extremity of latus rectum. No. 92 24–25
Planets, being supposed to have been originally one mass, are separated by an explosion, and force varying directly as distance; to prove the simultaneous colision of the planets. No. 90 22–24
Principal moment of inertia of triangle at centroid. No. 99 270–271
Rope passes over pulley, having at one end a given weight, and at other end hangs a monkey of a given weight. Find condition of impossibility of the mankey's climbing to the pulley. No. 93 48
Spool placed on a rough table. Vid. problem. No. 98 196
Two particles, subject to mutual attraction, and fixed center, move in a plane containing that center. Find the motion under the law of inverse square. No. 96 174–175

DIOPHANTINE ANALYSIS.

Arrange the consecutive integers 1 to n^2 as a magic square, when n is odd. Apply when $n=9$. No. 75 25–26
Four positive numbers to be found such that if each be diminished by twice the cube of their sum the four remainders will be rational cubes. No. 76 49–50
In series $1^3+3^3+5^3+\ldots$ find n so that the nth term and sum of n terms shall both be squares. No. 82 166
Integral right triangle in which the bisector of one of the acute angles is always integral. No. 79 83–85
Three square numbers in harmonic progression. No. 78 82–83
Three square numbers whose reciprocals form an arithmetical progression. No. 80 112–113

AVERAGE AND PROBABILITY.

Average area of a quadrilateral formed by joining the extremities of two chords perpendicular to each other and passing through a point at distance a from center of a circle radius R, No. 82 52–53
Average area of all ellipses of given major axis, No. 83 53–54
Box contains 100 balls marked from 1 to 100. 13 balls are drawn. Find chance that balls marked from 1 to 10 are among the 13. No. 80 27
From a point in the circumference of a circle two chords are drawn; find (1) average radius; and (2) averape area of circle touching the two chords and given circle. No. 84 113–114
Inch auger-hole bored through sphere. Find average volume of hole. No. 89, 197–198
Mean distance of all points on side of equilateral triangle from opposite vertex, and average length of line drawn across triangle. No. 81 50–52
Mean distance of random point in a sphere from a point, first, within; second, without, sphere. No. 87 141
Pond, circular, formed during a dark night in a circular field. Find chance of man walking into pond 296
Tetrahedron, average volume of, formed by four random points in a sphere. No. 88 176–177
Two equal spheres touch each other externally. If a point be taken within each sphere, show that, first, the distance between the points is less than diameter of either sphere is 13/55, and, second, the average distance between them, 11/52. No. 79 26–27
Two points are taken at random in a circle and a chord drawn through them; a point is taken at random in each segment. Find average area of

quadrilateral formed by joining the four points. No. 85 114–115
Two points are taken at random in a circular annulus of two concentric circles. Find chance that the straight line joining the two points will not cut the inner variable circle. No. 86 140–141

MISCELLANEOUS.

Cask, in shape of frustrum of spheroid, lies in a horizontal position. Find quantity of water in it. No. 81 178–179
Base-ball pitcher's curves to be explained. No. 77 142
Center of a regular polygon of n sides moves along a diameter of circle, its magnitude varying so that one of diagonals always coincides with a chord. Find volume and surface generated. No. 78 142–143
Ellipse seen as circle. No. 74 ..... 54-55, 115–116
Ice cream freezer, In an, cream of a homogeneous, etc., vid. problem. No. 73 36
Latitude λ°, In, a tree$=a$ leans in a direction $=\beta$, with an angle of elevation$=\gamma$. The sun's declination$=\delta$. Find direction of shadow when sun is on meridian. No. 79 144–145
Notes, Two, one for a dollars at $m\%$, etc., vid. problem. No. 86 272
Prove $e^{\pi} + e^{-\pi} = 2[1+2^2][1+(\frac{2}{5})^2]$. . . ad infinitum. No, 84 199–200
Rhombuses, Rational, wherein area is by 1 less than the square of its sides. No. 80 145
Rational right triangle has at least one side divisible by 5. No. 85 ..... 232-233, 271
Segment's area of a circle being given, find chord. No. 83 199
Show $\log[x-a-b\sqrt{-1}] = \frac{1}{2}\log[(x-a)^2+b^2] - \sqrt{-1}.\tan^{-1}[b/(x-a)]$ No. 76 118–119
Spheres, Two equal, in contact. Find magnitude of sphere in contact with them. No. 82 232
Water tank at the Nacogdoches River, etc., vid. problem. No. 75 116–118

THE AMERICAN MATHEMATICAL MONTHLY.

DEVOTED TO THE

SOLUTION OF PROBLEMS IN PURE AND APPLIED MATHEMATICS, PAPERS ON MATHEMATICAL SUBJECTS, BIOGRAPHIES OF NOTED MATHEMATICIANS, ETC.

EDITOR:

B. F. FINKEL, A. M., M. Sc.,

MEMBER OF THE LONDON MATHEMATICAL SOCIETY; MEMBER OF THE AMERICAN MATHEMATICAL SOCIETY, AND PROFESSOR OF MATHEMATICS AND PHYSICS IN DRURY COLLEGE, SPRINGFIELD, MISSOURI.

ASSOCIATE EDITOR:

JOHN M. COLAW, A. M.,

MEMBER OF AMERICAN MATHEMATICAL SOCIETY, MONTEREY, VIRGINIA.

DIXON BROS. PRINTERS, 233 EAST COMMERCIAL STREET, SPRINGFIELD, MO.